

Analysis of parameter identification methods for electrical Li-Ion battery modelling

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Abstract

To fulfil lifetime requirements of a HV-Battery by simultaneously keeping drive performance it is indispensable to well-suit the BMS application. Therefore, knowledge of battery aging and the change of the battery model parameters in time are mandatory. Introducing a real time Li-Ion battery model, this article issues the topic of parameter identification and which kind of optimization method fits the best to the optimization problem on hand for stable and fast parameter identification.

Keywords: Li-Ion battery model, parameter identification, particle swarm optimization, gradient method, impedance

1 Introduction

Considering battery aging, it is a challenge to an Electric-/ Hybrid-Vehicle's Battery Management System (BMS) to guarantee the lifetime requirement of 10 to 15 years. For this purpose aging tests are done to get knowledge about the mechanisms of calendric and cyclic aging with the goal to derive models describing this behaviour. After that, an existing electro-thermal battery model at Begin of Life (BOL) can be extended to the influence of aging. Such a model allows long term offline simulation until a certain End of Life (EOL) criterion to determine the best BMS application to a specific operation strategy. To reach this final model state, many investigations have to be performed usually in connection with the issue of parameter identification. Therefore, parameter identification of an electrical battery model is a fundamental topic. These parameters are extracted from measured data using a well-defined parameterization test. In reference to aging, it is important to learn how model parameters change in time. For this purpose, aging tests are

interrupted at specified states to perform parameterization tests.

Introducing a battery model for automotive purposes, this article shows the history of how an optimization algorithm was found for the given optimization problem on hand and knowledge gained therein. Parameterization tests and existing connection to parameter identification are not issued but will be discussed in more detail in a separate paper.

This paper is structured as follows. Sections two and three give insight in the modelling approach and mathematical description. Section four issues the steps from a general to an optimization problem specific parameter identification method.

2 Modelling Approach

A very common technique for battery state description is impedance spectroscopy (EIS). Figure 1 shows an example of a battery's impedance curve.

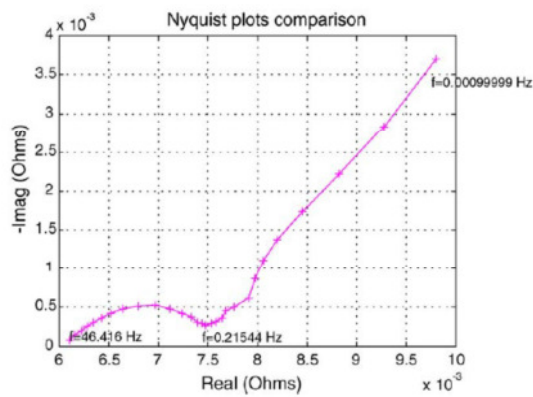


Figure 1: Example of an impedance curve [3]

A typical diffusion characteristic is the line for lower frequencies. A value, which represents diffusion processes within a cell, is the so called Warburg Impedance and is defined for Laplace domain only. Sometimes a Constant Phase Element (CPE) is used instead of the Warburg impedance. In literature, Li-Ion battery models in Laplace- or Fourier domain description can be found frequently. Depending on assumptions like flat or porous electrodes [1, 2], the models are more or less complex. Nevertheless, a wide spread approach to battery modelling is the use of Equivalent Electric Circuits (EEC). Figure 2 shows an example of a possible EEC model. It consists of an ohmic resistance $R\Omega$, a RC pair and the Warburg impedance $Zw(s)$ in series. The RC pair describes double layer capacity Cdl and transfer resistance Rtc effects within a battery cell.

For automotive applications it is crucial to find model approaches that can be computed online on control units. This excludes models in Laplace domain. So the mathematical description of Warburg impedance in time domain is the only problem that prevents the usage of EEC so far. Possible solutions are presented in [3] by introducing Cauer- and Foster Networks to model the diffusion process, Figure 3 and Figure 4.

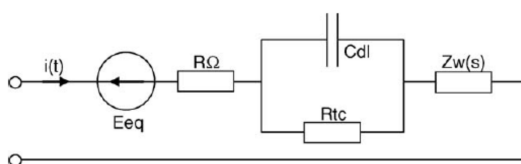


Figure 2: Equivalent Electric Circuit model in Laplace space including Warburg Impedance $Zw(s)$ [3]

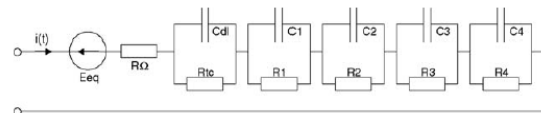


Figure 3: Example using a Foster Network [3]

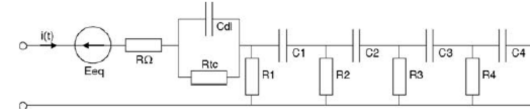


Figure 4: Example using a Cauer Network [3]

Through simple usage of several RC-pairs, it is possible to fit the model to measured EIS curve as close as necessary. Figure 5 shows such a fit and reveals that it is a free choice which kind of network to use as Cauer and Foster curves coincide. However, in this paper a Foster Network is preferred. In conclusion, the modelling approach is an EEC consisting of an Open Circuit Voltage (OCV), an ohmic resistance R_0 and several RC pairs ($R_j || C_j$) in series, see Figure 6.

It is very important to highlight that building up a battery model for simulation in respect to automotive purposes, there is a difference to EIS curve modelling. As EIS is a description of small signal behaviour (criteria for linearity are not hurt) some interpretation of model parameters to physical phenomena like double layer capacity etc. is allowed. The model introduced in this paper is a description of large signal behaviour of a highly nonlinear system. For this reason, value interpretation is hardly possible.

As the presented approach is valid for a single cell as well as a battery pack, within this paper the term "cell" also represents battery modelling. For correct understanding the contact resistances are usually comprised in R_0 .

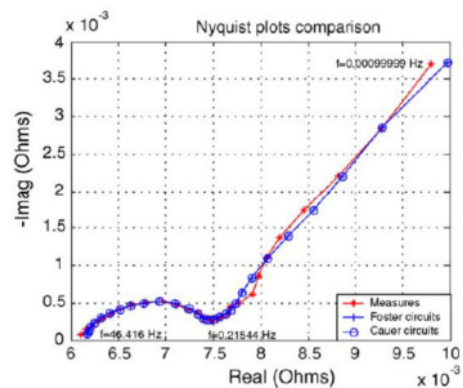


Figure 5: Comparison of Nyquist plots [3] (red-measured, blue-fitted)

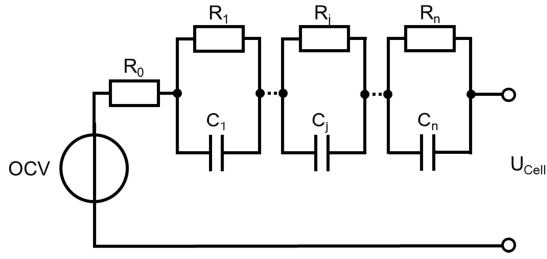


Figure 6: Final EEC modelling approach

3 Mathematical description of a Li-Ion Battery Model for automotive purpose

The main quantities that have influence on the parameters are:

- State of Charge (*SOC*)
- Temperature (θ)
- Current direction, i.e. charge or discharge (*sign(I)*) and
- Current magnitude (*abs(I)*) which can be expressed in C-Rates

The computation of C-Rate is simple as it is current normalized to nominal cell capacity.

From analysis of measured data, there is a model that sufficiently describes a cell's voltage response to current input. Given a model with an ohmic resistance R_0 and n RC pairs with R_j and $\tau_j = R_j C_j$ the parameters have following dependencies:

- $R_0 = f(SOC, \theta, sign(I), abs(I))$
- $R_j = f(SOC, \theta, sign(I))$
- $\tau_j = f(SOC, \theta)$

A comfortable way for mathematical description is the state space form. Equations (1) and (2) show the typical continuous state space formulation.

$$\dot{x} = A \cdot x + B \cdot i \quad (1)$$

$$\hat{u} = C \cdot x + D \cdot i \quad (2)$$

For each RC-pair in Figure 6 an ordinary differential equation according to (3) is valid.

$$\tau_j \cdot \dot{x}_j + x_j = R_j \cdot i \quad (3)$$

In (3) variable x_j stands for the voltage at the j -th capacitor and variable i stands for cell current. The overall voltage drop \hat{u} is the sum of the ohmic voltage drop u_0 and all transient voltages (4).

$$\hat{u} = u_0 + \sum_{j=1}^n x_j \quad (4)$$

Transformation of equations (3) and (4) into (1) and (2) results in a state space structure with canonical parameterization form. The matrices are as follows:

$$A = \begin{bmatrix} -1/\tau_1(SOC, \theta) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & -1/\tau_n(SOC, \theta) \end{bmatrix}$$

$A \in \mathbb{R}^{n \times n}$

$$B = \begin{bmatrix} R_1(SOC, \theta, sign(I)) / \tau_1(SOC, \theta) \\ \vdots \\ R_n(SOC, \theta, sign(I)) / \tau_n(SOC, \theta) \end{bmatrix}$$

$B \in \mathbb{R}^{n \times 1}$

$$C = [1 \quad \dots \quad 1]$$

$C \in \mathbb{R}^{1 \times n}$

$$D = R_0(SOC, \theta, sign(I), abs(I))$$

$D \in \mathbb{R}$

The full mathematical formulation ends up in a Linear Parameter Varying (LPV) system. The number of parameters needed to be identified is large and depends on the model order n as well as on selected nodes for temperature, SOC and C-Rates. Furthermore, a parameterization test must be defined that takes all four quantities into account equally. For OCV measurement, specified temperatures and SOC nodes are run on a test bench. This helps to simplify the mathematical model for identification purpose as a parameterization test can follow an OCV measurement. So matrices A, B and D get a simplified structure at a certain SOC and temperature node:

$$A = \begin{bmatrix} -1/\tau_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & -1/\tau_n \end{bmatrix} \quad A \in \mathbb{R}^{n \times n}$$

$$B = \begin{bmatrix} R_1(\text{sign}(I))/\tau_1 \\ \vdots \\ R_n(\text{sign}(I))/\tau_n \end{bmatrix} \quad B \in \mathbb{R}^{n \times 1}$$

$$D = R_0(\text{sign}(I), \text{abs}(I)) \quad D \in \mathbb{R}$$

To consider current direction, it is necessary to split the row current vector i in Charge (Ch) and Discharge (DCh) according to (5). Theoretically, this transformation results in a system with two input signals.

$$i = \begin{bmatrix} i_{Ch} \\ i_{DCh} \end{bmatrix} \quad (5)$$

In the next step matrices B and D needed to be transformed to current direction dependency according to (6) and (7), too.

$$B = [B_{Ch} \quad B_{DCh}] = \begin{bmatrix} R_{1,Ch}/\tau_1 & R_{1,DCh}/\tau_1 \\ \vdots & \vdots \\ R_{n,Ch}/\tau_n & R_{n,DCh}/\tau_n \end{bmatrix} \quad (6)$$

$$D = [R_{0,Ch} \quad R_{0,DCh}] \quad (7)$$

Finally, C-Rate dependency of D has to be taken into account. Up to now no specific equation is known to the author that describes this behaviour. For this reason a linear spline function is chosen as a general and at the same time simple approach. By that, the elements of vector D in (7) can be expressed in spline representation according to equations (8) and (9)

$$R_{0,Ch} = R_{0,Ch}^* \cdot \left(1 + \sum_{q=1}^m a_q \cdot Bf_q \right) \quad (8)$$

$$R_{0,DCh} = R_{0,DCh}^* \cdot \left(1 + \sum_{q=1}^m c_q \cdot Bf_q \right) \quad (9)$$

Expression Bf in (8) and (9) stands for the linear spline Base Function. Equation (10) defines Bf .

$$Bf_q = \begin{cases} CRate - CRateNode_q & \text{if } CRate > CRateNode_q \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

Variable $CRate$ goes with current at a certain time step and variable $CRateNode$ is a defined C-Rate node. The number of C-Rate nodes is expressed by m . Within (8) and (9) a_q and c_q are spline coefficients. Table 1 gives a summary of parameters needed to be identified for a certain temperature and SOC node. The amount of parameters can be calculated by $p = 3 \cdot n + 2 \cdot (m + 1)$. As an example, for a simple case of $n = 2$ and $m = 4$ the number of unknown parameters results in $p = 16$.

Table 1: Summary of parameters needed to be identified

time constants	$\tau_1 \quad \dots \quad \tau_n$
RC gain factors	$R_{1,Ch} \quad \dots \quad R_{n,Ch} ;$ $R_{1,DCh} \quad \dots \quad R_{n,DCh}$
linear spline function coefficients	$R_{0,Ch}^* ; a_1 \quad \dots \quad a_m ;$ $R_{0,DCh}^* ; c_1 \quad \dots \quad c_m$

4 Parameter Identification

Following subsections give insight into the optimization problem on hand.

4.1 Heuristic Method

If a new optimization problem is unknown it is appropriate to start with a general method for parameter identification to assure not getting stuck in a possible local minimum. For this reason, a heuristic method was chosen. Heuristic methods are working with particles, i.e. valid parameter sets, to search within a search space for a global best parameter set. These methods have more chances to find a global minimum between many local minima as all particles are spread randomly in a search space. This is also an advantage as no initial parameter values are necessary. Such algorithms are for instance Genetic Algorithms (GA) and Particle Swarm Optimizer (PSO). Because of its simplicity, PSO was chosen for first identification. The general PSO algorithm consists of two equations. With the velocity equation (11), a new velocity vector of a particle is computed. Therein w , c_1 and c_2 are weighting factors. To give some unforeseen behaviour, two uniformly

distributed numbers r_1 and r_2 are included. The current particle position is represented by X . Typical PSO characteristic is the fact that every particle remembers its best achieved position P . Additionally, there exists a global best position G of all particles.

$$v^* = w \cdot v + c_1 r_1 [P - X] + c_2 r_2 [G - X] \quad (11)$$

The second equation (12) updates a particle's position. Both equations are illustrated in Figure 7.

$$X^* = v^* + X \quad (12)$$

In this work a hybrid PSO (HPSO) as proposed in [4] was implemented. This algorithm includes aspects of GA and contributes a method to escape a possible local minimum. The scheme is shown in Figure 8. Starting with a parent population, a child 1 generation is computed with PSO equations (11) and (12). In parallel a child 2 generation is computed with Cauchy Mutation. This step shall help to escape a possible local minimum. After that, particles of the two child generations compete against each other in their fitness, i.e. the first particle of child 1 generation against the first particle of child 2 generation. This results in an offspring. Within the next step a new parent population is generated through a Natural Selection Strategy. Therefore, the old parent population is compared to the offspring generation. For more details refer to [4].

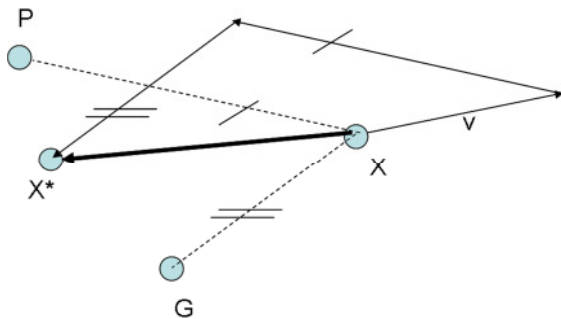


Figure 7: Illustration of PSO equations

Figure 9 shows a voltage response of an automotive cell to a discharge-charge current pulse. Red dashed line represents HPSO estimation of a battery model with $n = 2$ and $m = 0$, i.e. no C-Rate dependency. This example is used as reference for further discussion in the following sections.

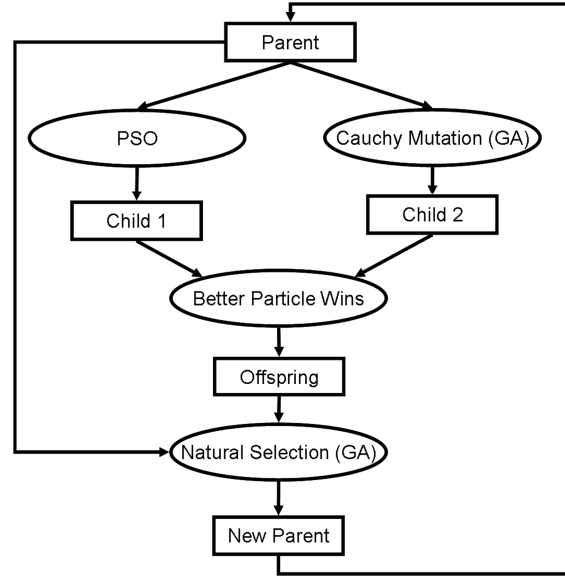


Figure 8: Scheme of HPSO algorithm

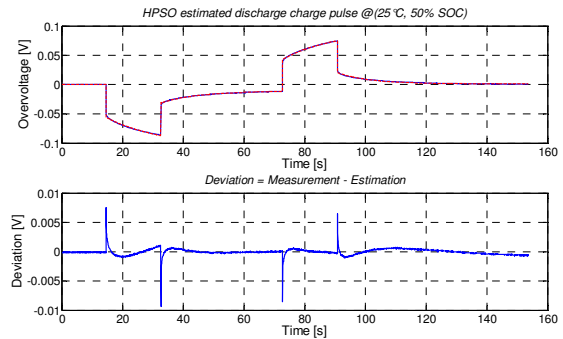


Figure 9: HPSO estimated discharge/charge pulse @ 25°C and 50% SOC

4.2 Grid Method

To check if HPSO is working properly, the best parameter set is additionally computed by a grid method. According to Table 2 a grid of about 20 million possible parameter sets is spanned across the defined parameter borders.

Table 2: Parameter grid borders

	$R_{1,Ch}$ [mΩ]	$R_{1,DCh}$ [mΩ]	$R_{2,Ch}$ [mΩ]	$R_{2,DCh}$ [mΩ]	τ_1 [s]	τ_2 [s]
Min:	0,1	0,0898	0,5	0,6	1	80
Max:	0,8	0,7184	3,7	4,6	40	176

For this study, the ohmic resistance was held at the HPSO identified value. Table 3 highlights that HPSO and grid method found the same parameter set. The slight disagreement can be explained by a rough grid. Resulting costs, which is the Sum of Squared Errors (SSE), are excellent.

Table 3: Comparison of results – HPSO vs. Grid

parameter	HPSO	Grid
$R_{1,Ch}$ [m Ω]	0,4003	0,4065
$R_{1,DCh}$ [m Ω]	03592	0,3648
$R_{2,Ch}$ [m Ω]	1,8263	1,8548
$R_{2,DCh}$ [m Ω]	2,2911	2,3269
τ_1 [s]	10	10
τ_2 [s]	147	152
Costs (SSE)	8,9177e-4	9,1325e-4

4.3 Sensitivity Analysis

This study shall highlight the influence of a varied parameter to a cost function. Furthermore, it can give a hint whether HPSO is the right tool for this kind of optimization problem. This is exemplarily shown for the obtained parameter set illustrated in Figure 9. Except the studied parameter, all other parameters were held at their HPSO- identified values. Herein SSE is defined as the objective function value. The result shown in Figure 10 and Figure 11 highlights that for a given model order a unique solution exists.

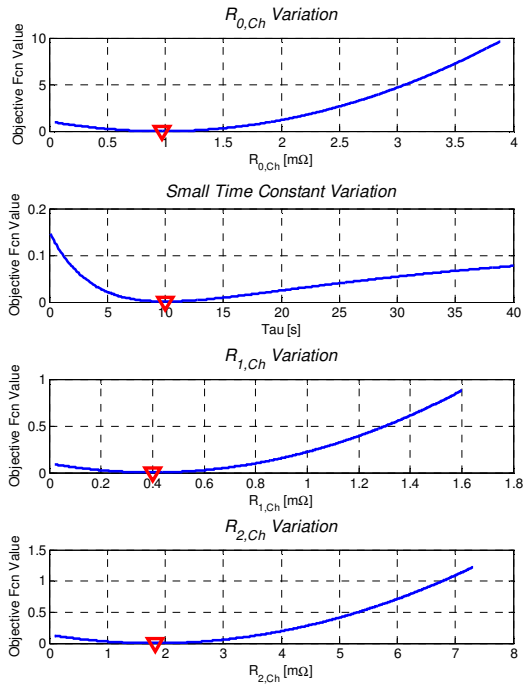


Figure 10: Sensitivity analysis

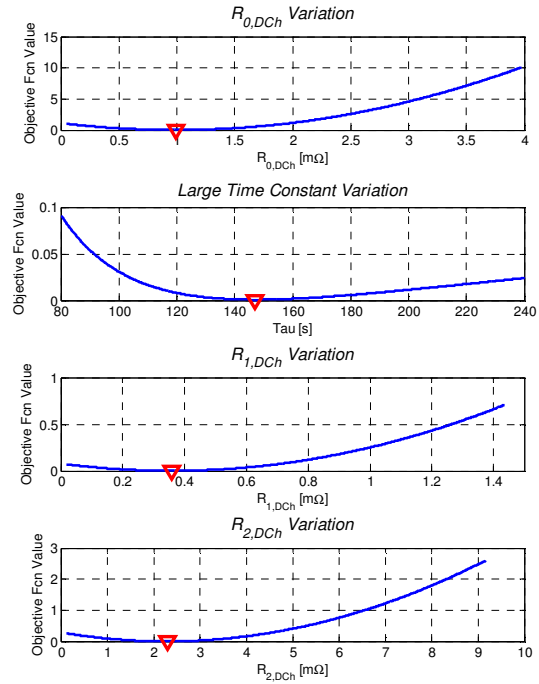


Figure 11: Sensitivity analysis (sequel to Figure 10)

It is remarkable that except a global minimum no further local minimum is observable. The red triangle points at the HPSO identified parameter value. The key results are as follows. If a parameter value is varied from its optimum, it results in an increase of SSE. If a parameter is allowed to change to 100% off its optimum value, ohmic resistance has the most significant influence on SSE. It is interesting to realize that time constants have little effect on SSE in a relative wide parameter range, for instance $\tau_1 \in [7, 14]$ and $\tau_2 \in [130, 170]$. It turned out that herein made knowledge is applicable to all identified examples of this kind conducted by the author.

4.4 Gradient Method

From the previous section it becomes obvious that for this particular optimization problem a gradient based optimizer is the right choice. As an advantage this would result in faster parameter identification, too. In [5] a gradient method for parameters in state space form is proposed and shall be explained shortly.

To estimate a model that fits measured data, the sum of squared errors (SSE) for N data samples (13) is defined as objective function.

$$J(\theta) = \frac{1}{2} \cdot \sum_{k=1}^N [u_k - \hat{u}_k(\theta)]^2 \quad (13)$$

Therein u_k represents k-th data sample and $\hat{u}_k(\theta)$ is a parameter set dependent estimation value for cell overvoltage. Cell overvoltage means that OCV is subtracted from the measured voltage according to given SOC-OCV curves. For computation of $\hat{u}_k(\theta)$ discrete state space, given by equations (14) and (15), is needed. The matrices A and B are transformed to discrete form A_d and B_d that depend on sample time.

$$x[k+1, \theta] = A_d(\theta) \cdot x[k] + B_d(\theta) \cdot i[k] \quad (14)$$

$$\hat{u}[k, \theta] = C \cdot x[k] + D(\theta) \cdot i[k] \quad (15)$$

Figure 12 shows the iterative steps of discussed gradient method. The first main step includes computation of minimizing direction $\mu(\theta)$. Starting with equation (16)

$$\mu(\theta) = \operatorname{argmin} \|\Psi(\theta)^T \mu - E(\theta)\|_2 \quad (16)$$

a stable minimizing direction in the sense of least square is given by (17) wherein the symbol \dagger denotes the pseudo inverse of a matrix.

$$\mu(\theta) = [\Psi(\theta)^T]^\dagger E(\theta) \quad (17)$$

The column vector $E(\theta)$ (18) is a deviation vector of measured and estimated values.

$$E(\theta) = \begin{bmatrix} u[1] - \hat{u}[1, \theta] \\ \vdots \\ u[k] - \hat{u}[k, \theta] \\ \vdots \\ u[N] - \hat{u}[N, \theta] \end{bmatrix} \quad (18)$$

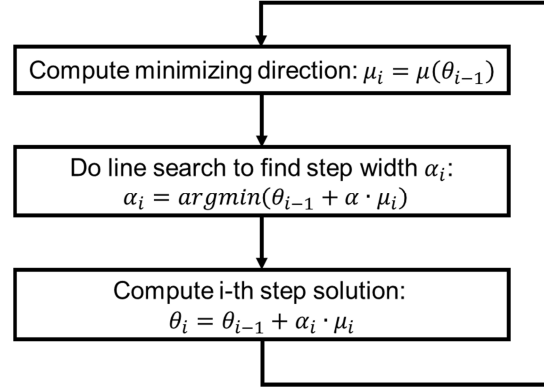


Figure 12: Scheme of gradient algorithm

The elements of matrix $\Psi(\theta)$ (19) are partial gradient values for each data sample, i.e. $\Psi(\theta) \in \mathbb{R}^{p \times N}$.

$$\Psi(\theta) = \begin{bmatrix} \psi(1, \theta_1) & \dots & \psi(k, \theta_1) & \dots & \psi(N, \theta_1) \\ \vdots & & \vdots & & \vdots \\ \psi(1, \theta_p) & \dots & \psi(k, \theta_p) & \dots & \psi(N, \theta_p) \end{bmatrix} \quad (19)$$

For computation of $\Psi(\theta)$ it is necessary to partial differentiate equations (14) and (15) with respect to a certain parameter (20). Considering the product rule for differentiation, partial derivation of discrete state space leads to equation (21) and (22).

$$\psi_l(k, \theta) = \frac{\partial \hat{u}(k, \theta)}{\partial \theta_l} \quad (20)$$

$$\frac{\partial x[k+1]}{\partial \theta_l} = A_d \frac{\partial x[k]}{\partial \theta_l} + A'_d x[k] + B'_d i[k] \quad (21)$$

$$\psi_l(k, \theta) = \frac{\partial \hat{u}[k, \theta]}{\partial \theta_l} = C \frac{\partial x[k]}{\partial \theta_l} + C' x[k] + D' i[k] \quad (22)$$

After computing equations (14), (15), (21) and (22), matrix $\Psi(\theta)$ can be built up. Finally, minimizing direction $\mu(\theta)$ is received from equations (18) and (17). The second main step of the proposed gradient method is a line search along the computed minimizing direction. Some possible line search algorithms can be found in [6]. Starting with a parameter set from previous iteration the

goal of a line search is to find a step width α along a given minimizing direction μ , for which the objective function gets a minimum. Having found a step width, the parameter set is updated according to equation (23) and a new iteration follows as long as a stopping criterion is not fulfilled.

$$\theta_i = \theta_{i-1} + \alpha \cdot \mu \quad (23)$$

In Table 4 HPSO identified parameters are compared to those identified by the new gradient method. The parameter values are similar. It is remarkable that the large time constant has a higher value and is within the interval mentioned in section 4.3. The gradient method even found a parameter set which costs are slightly better compared to HPSO. For demonstration Figure 13 shows an example of C-Rate dependent ohmic resistance ($m = 32$) identified from measured parameterization test data.

Table 4: Comparison of results – HPSO vs. Gradient

parameter	HPSO	Gradient
$R_{0,Ch}$ [m Ω]	0,972	0,975
$R_{0,DCh}$ [m Ω]	0,993	0,993
$R_{1,Ch}$ [m Ω]	0,4003	0,4041
$R_{1,DCh}$ [m Ω]	03592	0,3723
$R_{2,Ch}$ [m Ω]	1,8263	2,0919
$R_{2,DCh}$ [m Ω]	2,2911	2,5289
τ_1 [s]	10	10
τ_2 [s]	147	169
Costs (SSE)	8,9177e-4	8,90e-4

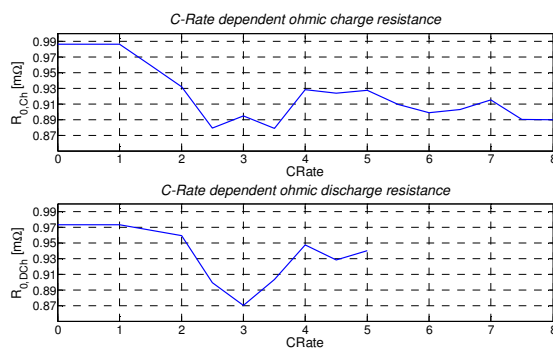


Figure 13: Example for C-Rate dependent ohmic resistance pulse @ 25°C and 50% SOC

5 Conclusion

This paper introduces a battery model for automotive purposes. For parameterization, a suitable optimization method is needed for parameter identification. These parameters are

extracted from measured data of a well-defined parameterization test. Starting with a general heuristic parameter identification approach, it is shown that a gradient method is suitable for the optimization problem on hand as a unique solution for a given model order exists.

To fulfil lifetime requirements of a HV-Battery by simultaneously keeping drive performance it is indispensable to well-suit the BMS application. Knowledge is mandatory of how the battery is aging. For this reason, long term aging tests are done. One important point of interest is the change of model parameters in time. For such a study a parameterization test is carried out during aging tests regularly. The proposed gradient method is a very efficient approach for parameter identification out of these measured data. However, gaining knowledge about battery aging gives the opportunity to derive aging models. These can be connected to existing electro-thermal battery models for long term offline system simulation to find a suitable BMS application for a certain operation strategy.

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